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Weak-Lensing by Large-Scale Structure and the Polarization Properties of Distant Radio-Sources

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ABSTRACT

We estimate the effects of weak lensing by large-scale density inhomogeneities and long-wavelength gravitational waves upon the polarization properties of electromagnetic radiation as it propagates from cosmologically distant sources. Scalar (density) fluctuations do not rotate neither the plane of polarization of the electromagnetic radiation nor the source image. They produce, however, an appreciable shear, which distorts the image shape, leading to an apparent rotation of the image orientation relative to its plane of polarization. In sources with large ellipticity the apparent rotation is rather small, of the order (in radians) of the dimensionless shear. The effect is larger at smaller source eccentricity. A shear of 1% can induce apparent rotations of around 5 degrees in radio sources with the smallest eccentricity among those with a significant degree of integrated linear polarization. We discuss the possibility that weak lensing by shear with rms value around or below 5% may be the cause for the dispersion in the direction of integrated linear polarization of cosmologically distant radio sources away from the perpendicular to their major axis, as expected from models for their magnetic fields. A rms shear larger than 5% would be incompatible with the observed correlation between polarization properties and source orientation in distant radio galaxies and quasars. Gravity waves do rotate both the plane of polarization as well as the source image. Their weak lensing effects, however, are negligible.

Subject headings: cosmology: theory — gravitational lensing — large-scale structure of universe — polarization

1. Introduction

Polarization properties of extragalactic radio sources are primarily a source of information about magnetic fields in the source, in the intergalactic medium, and in the Milky Way (Saikia & Salter 1988, Kronberg 1994). The orientation of the plane of linear polarization at the source, derived from multi-frequency polarization measurements after correction for Faraday rotation

along intervening magnetic fields, should point perpendicularly to the magnetic field lines, the emission being synchrotron radiation.

In the case of extended radio jets from quasars, the orientation of the intrinsic linear polarization within a well defined jet is typically perpendicular to the local direction of each jet segment, and if it does change it is usually a 90° flip (see, e.g., Bridle, Perley & Henriksen 1986; and Killen, Bicknell, & Ekers 1986). Computational MHD modeling of radio jets improves the confidence in the predicted jet-intrinsic magnetic field configurations supporting these observations (Clarke, Norman, & Burns 1989).

Kronberg et al. (1991) have proposed and applied a technique involving the angular relationship between the intrinsic polarization vectors and the morphological structure of extended radio jets to probe the gravitational distortion of these sources by foreground lensing galaxies. Since the polarization vector is not rotated by the gravitational field of an ordinary gravitational lens, the bending of a jet away from its intrinsic projected shape caused by the lensing effects of intervening masses yields a detectable local "alignment breaking" between the polarization and the orientation of each jet segment. Application of this idea to the analysis of the radio and deep optical images of the strongly polarized jet from 3C 9 provided the first estimate of the mass and mass distributions of two intervening galaxies acting as lenses using this technique (Kronberg et al. 1991; and Kronberg, Dyer, & Röser 1996).

The notion that the plane of polarization is not rotated by the gravitational field of an ordinary gravitational lens, which underlies the method of Kronberg et al. (1991), was addressed by Dyer & Shaver (1992). Using symmetry considerations they argued that the polarization direction of photons in a beam under the influence of a gravitational lens remains unaffected except for lenses in very rapid rotation, an astrophysically unlikely situation. Similar conclusions were also reached by Faraoni (1993), with a different approach. The fact that the gravitational field of rotating bodies may rotate the plane of polarization of electromagnetic waves has been addressed among others by Strotskii (1957), Balazs (1958), Plebanski (1960), Mashhoon (1973), Pinneault & Roeder (1977) and Su & Mallet (1980).

In the present work we analyse the effect of weak gravitational lensing by large scale structure in the Universe upon the polarization properties of electromagnetic radiation from distant sources. Through an extension of the method of "alignment breaking", introduced by Kronberg et al. (1991), we suggest that polarization properties of distant radio sources may provide information on the fluctuations in the gravitational potential due to large scale density inhomogeneities.

When light from a distant source propagates across density inhomogeneities, the apparent shape and size of the source is distorted by tidal deflections. The effect is known as weak lensing, in contrast to the case when the bending of light rays is so strong that multiple image formation is possible. Weak lensing effects have already proved to be a powerful tool to map the mass distribution in galactic halos (Tyson et al. 1984) and clusters of galaxies (Tyson, Valdes, & Wenk 1990). They are also a potential probe of density fluctuations on the largest scales (Blandford

et al. 1991, Miralda-Escudé 1991, Kaiser 1992), particularly through the correlation in galactic ellipticities induced by the shear they produce. It has been estimated that shear caused by large-scale density fluctuations in the linear regime is of the order of 1%, depending on the cosmological model, the normalization of the power spectrum, and the redshift distribution of sources (Blandford et al. 1991, Miralda-Escudé 1991, Kaiser 1992). Theoretical predictions on smaller scales (below ∼20 arcmin) by Jain & Seljak (1996), show that density inhomogeneities in the non-linear regime induce a larger cosmic shear, of the order of a few percent. Recently, Schneider et al. (1997) presented the first evidence for the detection of a robust shear signal of about 3% on scales of 1 arcmin, in agreement with the above theoretical expectations, from the analysis of data in a field containing the radio source PKS1508-05. Long-wavelength gravitational waves (tensor metric perturbations) are also a potential source of weak lensing of distant sources.

Radio sources with a significant degree of linear polarization, are also usually elongated along one direction. The angle $\chi - \psi$ between the intrinsic plane of polarization and the orientation of the structural axis has been measured for a large number of sources, for which redshift information is also available. The existence of a significant correlation between the direction of integrated linear polarization and the source axis orientation has been noticed since the earliest investigations (Haves & Conway 1975, Clarke et al. 1980). For high redshift galaxies, a well defined peak is found around $\chi - \psi = 90°$, suggesting a magnetic field parallel to the source axis. For low redshift sources there is weaker correlation, with a narrow peak at $\chi - \psi = 0°$, suggesting a magnetic field perpendicular to the source axis, and a broader peak at $\chi - \psi = 90°$. It is believed that the $\chi - \psi = 90°$ peak corresponds to high-luminosity sources, while the $\chi - \psi = 0°$ peak is ascribed to low-luminosity objects.

The angle between the plane of integrated linear polarization and the major axis of distant radio sources is potentially a useful tool to search for unusual effects upon light propagation over cosmological distances, additional to the standard Faraday rotation. There have been, for instance, searches for correlations between the polarization properties of extragalactic radio sources and their location, which would evidence some kind of "cosmological birefringence". While first attempts indicated that the residual polarization rotation fitted a dipole rule (Birch 1982, Kendall & Young 1984, Bietenholz & Kronberg 1984), a later analysis of a larger sample by Bietenholz (1986) rejected that evidence. Recently, Nodland & Ralston (1997) have claimed to find evidence in the data for a redshift-dependent dipole anisotropy in the rotation of the plane of polarization. A more conventional interpretation and statistical analysis of the same data (Carroll & Field 1997, Eisenstein & Bunn 1997) rejects any statistically significant positive signal, which would also be in conflict with other potentially more accurate probes of such cosmological rotation of polarization, through high resolution polarization and intensity data of some specific radio sources (Leahy 1997, Wardle et al. 1997).

In this paper, we estimate the effect of weak lensing by large scale structure, in the form of density fluctuations or gravitational waves, upon the observed angle between the plane of polarization of the electromagnetic emission by distant radio sources and their image orientation.

We find it is in principle possible that the scattered cases of distant radio sources which appear to be polarized away from the direction either perpendicular or parallel to the source axis, if Faraday rotation were the only propagation effect, be actually a consequence of additional wavelength-independent rotation of the polarization as their light travels to us through an inhomogeneous universe.

In section 2 we review the formalism appropriate to describe image distortions caused by scalar and tensor weak lensing. In section 3 we work in the geometric optics approximation and derive the rotation of the plane of polarization as light propagates in the presence of metric fluctuations. In section 4 we estimate the effect of large scale structure upon the angle $\chi - \psi$ between the plane of polarization and the orientation of a source image, and show that a shear of 1% can change the value of $\chi - \psi$ by 5°. In section 5, we discuss the possibility that scalar shear with a rms value around or below 5%, which would cause apparent rotations with a mean square dispersion of 20°, be the cause of the scattered cases of cosmologically distant radio sources with a direction of integrated linear polarization which is not perpendicular to their major axis. In section 6 we summarize our conclusions.

2. Image distortion by random metric perturbations

In this section we review the formalism appropriate to describe weak lensing effects upon light propagation along cosmological distances through a universe with large scale density inhomogeneities and long wavelength gravitational waves. Part of the material in this section is not new (see for instance Kaiser 1992, Seljak 1994, Kaiser & Jaffe 1997 and Kaiser 1996). We include it for completeness, to define our method and conventions, and because some of our results for the gravitational distortions are at variance with those of Seljak (1994) and Kaiser & Jaffe (1997). The differences do not conflict with the main conclusion that weak lensing effects by large scale density inhomogeneities accumulate over distances longer than the wavelength of the metric perturbation as $(D/\lambda)^{3/2}$ (Seljak 1994), while weak lensing distortions by gravitational waves do not significantly accumulate (Kaiser & Jaffe 1997). Even if the effect of gravitational waves is small, it is relevant to precisely determine the focusing, shear and rotation they induce if we want to determine the rotation of the plane of polarization relative to the morphological structure of the source.

We restrict our attention to effects that are linear in the metric perturbations. For simplicity, we shall consider scalar and tensor metric fluctuations around a Minkowski background. The generalization to a Robertson-Walker background is straightforward (Kaiser 1996). The restriction to a Minkowski background oversimplifies the distance-dependence of the effects under consideration, but preserves their qualitative features as well as the order of magnitude estimates. Vector perturbations could also be easily incorporated into this formalism.

Consider a Minkowski background metric $\eta_{\mu\nu}$ with signature (-1,1,1,1), and linear

perturbations $h_{\mu\nu}$. Greek indices $\mu\nu, \alpha, \beta, ... = 0, 1, 2, 3$ denote space-time coordinates. Latin indices $i, j, ... = 1, 2, 3$ will denote spatial coordinates, but latin indices $a, b, ... = 1, 2$ will be reserved for the components transverse to the unperturbed photons paths, that we shall take as approximately parallel to the x^3-axis. Boldface symbols will denote spatial vectors, i.e. $\mathbf{m}$, $\mathbf{k}$,...

The linearized geodesic equations read,

$$\frac{d^2 x^\mu}{dz^2} = -\eta^{\mu\nu}(h_{\nu\alpha,\beta} - \frac{1}{2}h_{\alpha\beta,\nu})p^\alpha p^\beta \quad . \tag{1}$$

We denote the photons 4-momentum with $p^\mu = dx^\mu/dz$, where z is an affine parameter (not necessarily coincident with the x^3-coordinate).

We consider photon paths sufficiently close to the x^3-axis, pointing towards an observer located at the origin of coordinates, and work not only up to linear order in the metric perturbations, but also up to first order in the photon departures away from the polar axis, which we parametrize as

$$x^a(z) = \theta^a z + O(h, \theta^2) \qquad (a = 1, 2) \quad . \tag{2}$$

We choose an affine parameter such that $x^3 = z + O(h, \theta^2)$, and thus $t = t_e + (z_s - z) + O(h, \theta^2)$, with t_e the time at which the photon is emitted at the source, located at an affine distance z_s in the x^3-direction. We assume a gauge such that $h_{0j} = 0$. To lowest order $p^\mu = (-1, \theta^1, \theta^2, 1)$, and the geodesic equation for the transverse departure of the photon path away from the x^3-axis can be written as:

$$\frac{d^2 x^a}{dz^2} = D^a(\mathbf{x}, t) + F^a{}_b(\mathbf{x}, t)\theta^b + O(\theta^2) \tag{3}$$

where

$$\begin{aligned} D_a &= \tfrac{1}{2}(h_{00} + h_{33})_{,a} + h_{a3,0} - h_{a3,3} \\ F_{ab} &= h_{ab,0} - h_{ab,3} - (h_{3a,b} - h_{3b,a}) \quad . \end{aligned} \tag{4}$$

Taylor-expanding $D_a(\mathbf{x}, t)$ to first order in x^a we can finally write

$$\frac{d^2 x^a}{dz^2} = D^a(z) + M^a{}_b(z)\theta^b + O(\theta^2) \tag{5}$$

where we defined

$$M_{ab}(z) = z D_{a,b}(z) + F_{ab}(z) \quad . \tag{6}$$

The quantities in the r.h.s of equation (5) are evaluated over the x^3-axis and along the unperturbed photon path. For instance $D_a(z) = D_a(x^a = 0, z, t = t_e + z_s - z)$.

In order to determine the distortion produced by metric fluctuations upon light propagation, we consider two neighboring photon paths that arrive to the observer with angular separation θ^a coming from two different points at the source separated by a coordinate distance $\Delta x^a(z_s) = \bar{\theta}^a z_s$. The particular solution to the geodesic deviation equation with the appropriate boundary conditions is

$$\Delta x^a(z) = \theta^a z + \int_0^z dz' \int_0^{z'} dz'' M^a{}_b(z'')\theta^b \quad . \tag{7}$$

A source with a shape described in these coordinates by $\Delta x^a = \bar{\theta}^a z_s$ is seen by an observer at the origin as if the image were at $\Delta \bar{x}^a = \theta^a z_s$ in the source sphere. The mapping $\bar{\theta}^a \to \theta^a$ is given by

$$\theta^a = (\delta^a{}_b + \psi^a{}_b)\bar{\theta}^b \tag{8}$$

where

$$\psi_{ab} = -\int_0^{z_s} dz\, \frac{z_s - z}{z_s}\, M_{ab}(z) \tag{9}$$

is the distortion matrix, which describes the effect of linear metric perturbations upon the observed shape of an image.

We point out now what is the source of variance between our results and those by Seljak (1994) and Kaiser & Jaffe (1997). Their expressions for the geodesic deviation and ψ_{ab} coincide with eqs. (7) and (9) with $M_{ab} = zD_{a,b}$, rather than the complete expression in eq. (6). The difference is the term F_{ab}, which arises as a consequence that the photon paths must converge into the origin, and thus the unperturbed paths are not strictly parallel. The relevant observational situation is not exactly described by the distortion suffered by a bundle of initially parallel photon paths, but rather by a bundle that starting at the source focalize at the observer location. In other words, tidal deflections are not only due to the gradients of the gravitational potential in the directions transverse to the central ray in the bundle, but there is also a contribution of the gradient in the longitudinal direction, given that the rays in the bundle are not parallel. The longitudinal gradients, as we shall see, are significant in the tensor case only, which at any rate is not likely to have a large impact upon measurements. The precise result for the focusing, shear and rotation induced by gravity waves, however, must take them into account.

There are some subtleties in the definition of the distortion matrix to linear order in the perturbations that we now discuss. As defined by eq. (9), the distortion matrix relates coordinate (rather than proper) shapes. It is thus convenient to separate from ψ_{ab} the local distortion effects, due to the different proper length that a same coordinate segment has at the observer and source locations. We thus define $\tilde{\psi}_{ab}$ such that

$$\psi_{ab} = \tilde{\psi}_{ab} + \frac{1}{2}\Delta h_{ab} \quad . \tag{10}$$

Here $\Delta h_{ab} = h_{ab}(z_s, t_e) - h_{ab}(z = 0, t = t_o)$ is the difference in the metric perturbations between the events of emission of the light ray at time t_e from the source location, and of its observation at the origin at time t_o. $\tilde{\psi}_{ab}$ is the appropriate matrix to describe the image distortion in terms of proper length mappings:

$$g_{ab}(0, t_o)\theta^a\theta^b\, z_s^2 = [g_{ab}(z_s, t_e) + \tilde{\psi}_{ab}]\; \bar{\theta}^a\bar{\theta}^b\, z_s^2 \tag{11}$$

with $g_{ab} = \eta_{ab} + h_{ab}$.

Although this is just a conventional matter, we also wish to emphasize that our definition of focusing describes its dependence with affine, rather than coordinate, distance. Since they differ by terms linear in the perturbations, this matter of convention needs to be specified.

The more general distortion matrix ψ_{ab} can be decomposed in terms of a diagonal matrix, which represents focusing or convergence, a symmetric traceless matrix γ_{ab}, which describes shear, and an antisymmetric matrix, that corresponds to a rigid rotation of the image:

$$\psi_{ab} = \kappa\,\delta_{ab} + \gamma_{ab} - \omega\epsilon_{ab} \quad . \tag{12}$$

Here ϵ_{ab} is the Levi-Civita tensor in two-dimensions. Shear can be described in terms of its intensity γ and the direction of one of its principal axis ϕ (which is only defined modulo π) as

$$\gamma_{ab} = \gamma \begin{pmatrix} \cos 2\phi & \sin 2\phi \\ \sin 2\phi & -\cos 2\phi \end{pmatrix} \quad . \tag{13}$$

Notice that a rotation of the direction of the shear by $\pi/2$ amounts to just a change in the sign of γ. Two independent directions in which to decompose an arbitrary shear form an angle $\pi/4$.

We now specialize the expressions above to the case of scalar and tensor metric perturbations respectively. We start working with a single planar metric fluctuation, and later compute the rms effect of a full stochastic background by superposition.

2.1. Scalar perturbations

We consider one Fourier mode of static scalar perturbations

$$h_{\alpha\beta}(\mathbf{x}) = h(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}\delta_{\alpha\beta} = h(\mathbf{k})e^{ik_a x^a}e^{-ik\mu z}\delta_{\alpha\beta} \tag{14}$$

appropriate for instance to describe adiabatic density perturbations in a matter-dominated, spatially-flat Robertson-Walker background, in the longitudinal gauge. We have defined $\mu = \mathbf{m}\cdot\mathbf{n}$, where μ is the cosine of the angle between the wave-vector $\mathbf{k} = k\mathbf{m}$, and the direction $\mathbf{n}$ in which light propagates, which is approximately along the x^3-axis, incoming towards the origin from positive values of x^3. Specialization of the eqs. in the previous section to this case lead to

$$\psi_{ab} = h(\mathbf{k})[\alpha(\mu, z_s)m_a m_b + \beta(\mu, z_s)\mu\delta_{ab}] \quad . \tag{15}$$

Here we defined

$$\begin{aligned} \alpha(\mu, z_s) &= k^2 \int_0^{z_s} dz \frac{z(z_s - z)}{z_s} e^{-ik\mu z} \\ \beta(\mu, z_s) &= -ik \int_0^{z_s} dz \frac{(z_s - z)}{z_s} e^{-ik\mu z} \quad . \end{aligned} \tag{16}$$

We write the projection of the wave-vector direction onto the plane orthogonal to z as $m^a = \sqrt{1-\mu^2}(\cos\varphi, \sin\varphi)$, and from eqs. (12) and (15) we find that the focusing, shear intensity and direction, and rotation induced by a single Fourier mode of scalar perturbations are

$$\begin{aligned} \kappa &= \tfrac{1}{2}h(\mathbf{k})[(1-\mu^2)\alpha(\mu, z_s) + 2\mu\beta(\mu, z_s)] \\ \gamma &= \tfrac{1}{2}h(\mathbf{k})\ (1-\mu^2)\alpha(\mu, z_s) \\ \phi &= \varphi \\ \omega &= 0 \quad . \end{aligned} \tag{17}$$

Scalar perturbations induce no rotation. They produce shear in the direction of the perturbation wave-vector, projected into the source plane.

As pointed out by Kaiser & Jaffe (1997), the reason why weak lensing effects by scalar perturbations accumulate over distances longer than the perturbation wavelength is evident from eqs. (16) and (17). Resonant modes, those for which the photons remain in phase with the perturbation, are those with $\mu \approx 0$, which produce a significant deflection. Notice that $\alpha(\mu) \to (kz_s)^2/6$ and $\beta(\mu) \to ikz_s/2$ as $\mu \to 0$. Our results differ from those by Seljak (1994) and Kaiser & Jaffe (1997) by the effect of the longitudinal modes which affect just the focusing κ by the term proportional to $\mu\beta(\mu, z_s)$. Since this term does not contribute in the resonant situation ($\mu = 0$), it is sub-dominant, and negligible, after superposition of all Fourier modes of the stochastic background.

As discussed in the previous subsection, the local effects can be subtracted from the distortion matrix describing proper distortions through $\tilde{\psi}_{ab} = \psi_{ab} - \frac{1}{2}\Delta h_{ab}$. In this case $\Delta h_{ab} = \delta_{ab}(e^{-ik\mu z_s} - 1)$, and use of the relation $2\mu\beta - \mu^2\alpha = (e^{-ik\mu z_s} - 1)$ allows to identify the proper distortion matrix elements as

$$\begin{aligned} \tilde{\kappa} &= \tfrac{1}{2}h(\mathbf{k})\alpha(\mu, z_s) \\ \tilde{\gamma} &= \gamma \quad . \end{aligned} \tag{18}$$

Consider now a stochastic background of scalar fluctuations characterized in momentum-space by the correlations $< h(\mathbf{k})h^*(\mathbf{k}') >= (2\pi)^3\delta(\mathbf{k} - \mathbf{k}')P(k)$ where $P(k)$ is the power spectrum. The mean square expectation value of the shear induced is

$$< \gamma^2 >= \int \frac{dk}{(2\pi)^3} k^2 P(k) \frac{\pi}{2} \int_{-1}^{1} d\mu \; (1 - \mu^2)^2 |\alpha|^2 \tag{19}$$

In the limit $kz_s \gg 1$ the μ-integral equals $\frac{\pi}{15}\ (kz_s)^3 + O[(kz_s)^2]$. Shear induced by scalar perturbations of amplitude h accumulates over distances D longer than the wavelength λ as $h \times (D/\lambda)^{3/2}$ (Seljak 1994). In the case of a scale invariant power spectrum $P(k)$, such as that predicted by inflationary models in a CDM scenario, with amplitude, normalized at small k according to the COBE-DMR measurement of the cosmic microwave background anisotropy, the rms shear is of order $< \gamma^2 >^{1/2} \approx 0.02$. The rms convergence is of the same order of magnitude.

2.2. Tensor perturbations

Now we study image distortions produced by gravity waves. We work tensor perturbations in the transverse traceless gauge, where $h_{\alpha\beta}k^\alpha = 0$, $h_{0\mu} = 0$, and $h^\mu_\mu = 0$. We consider gravitational waves with wave vector $k^\mu = k(1, \mathbf{m})$, with $m^i = (\sqrt{1-\mu^2}\cos\varphi, \sqrt{1-\mu^2}\sin\varphi, -\mu)$. As in the previous section, $\mu = \mathbf{m} \cdot \mathbf{n}$ is the cosine of the angle between the perturbation wave-vector and the photon propagation direction. We denote the two independent polarization modes of

the gravity waves with the symbols $+, \times$. The normal modes of the metric perturbations read $h_{ij}(\mathbf{x}, t) = h_{ij}(\mathbf{k}) e^{i(\mathbf{k}\cdot\mathbf{x} - kt)}$ with

$$\begin{aligned}
h_{11}(\mathbf{k}) &= h_+(\mathbf{k})(\mu^2 \cos^2\varphi - \sin^2\varphi) - h_\times(\mathbf{k})\mu \sin 2\varphi \\
h_{22}(\mathbf{k}) &= h_+(\mathbf{k})(\mu^2 \sin^2\varphi - \cos^2\varphi) + h_\times(\mathbf{k})\mu \sin 2\varphi \\
h_{33}(\mathbf{k}) &= h_+(\mathbf{k})(1 - \mu^2) \\
h_{12}(\mathbf{k}) &= h_+(\mathbf{k})\tfrac{1}{2}(1 + \mu^2) \sin 2\varphi + h_\times(\mathbf{k})\mu \cos 2\varphi \\
h_{13}(\mathbf{k}) &= h_+(\mathbf{k})\mu\sqrt{1-\mu^2} \cos\varphi - h_\times(\mathbf{k})\sqrt{1-\mu^2} \sin\varphi \\
h_{23}(\mathbf{k}) &= h_+(\mathbf{k})\mu\sqrt{1-\mu^2} \sin\varphi + h_\times(\mathbf{k})\sqrt{1-\mu^2} \cos\varphi \quad .
\end{aligned} \tag{20}$$

The distortion matrix now reads, from eq. (9)

$$\begin{aligned}
\psi_{ab} = \; & \alpha(\mu - 1, z_s)\left[\tfrac{1}{2} h_{33} m_a m_b - (1-\mu) h_{3a} m_b\right] - \\
& \beta(\mu - 1, z_s)\left[(1-\mu) h_{ab} + (h_{3a} m_b - h_{3b} m_a)\right]
\end{aligned} \tag{21}$$

where α and β are exactly the same functions as in eq. (16), now the argument being $\mu - 1$ instead of μ. We have discarded an irrelevant overall phase factor. The focusing, shear intensity and direction, and rotation induced by each normal mode of a gravity wave background, for each independent polarization, thus turn out to be

$$\begin{aligned}
\kappa &= \tfrac{1}{4}(1-\mu^2) h_+(\mathbf{k}) \left[(1-\mu)^2 \alpha(\mu-1, z_s) + 2(1-\mu)\beta(\mu-1, z_s)\right] \\
\gamma_+ &= \tfrac{1}{4} h_+(\mathbf{k}) \left[(1-\mu^2)(1-\mu)^2 \alpha(\mu-1, z_s) - 2(1+\mu^2)(1-\mu)\beta(\mu-1, z_s)\right] \\
\phi_+ &= \varphi \\
\gamma_\times &= \tfrac{1}{2} h_\times(\mathbf{k}) \left[(1-\mu^2)(1-\mu)\alpha(\mu-1, z_s) + 2\mu(1-\mu)\beta(\mu-1, z_s)\right] \\
\phi_\times &= \varphi - \tfrac{\pi}{4} \\
\omega &= -\tfrac{1}{2}(1-\mu^2) h_\times(\mathbf{k}) \left[(1-\mu)\alpha(\mu-1, z_s) + 2\beta(\mu-1, z_s)\right] \quad .
\end{aligned} \tag{22}$$

Gravity waves have quite different weak lensing effects than scalar metric perturbations. The fact that waves propagate imply that the resonant modes, those that may eventually add-up over distances longer than the wavelength, are now those with $\mu \approx 1$, which do not produce neither focusing, shear, nor rotation. Eqs. (22) coincide with the equivalent expressions of Kaiser & Jaffe (1997), except for the terms proportional to $\beta(\mu - 1, z_s)$.

The $+$ polarization of the gravity waves produces focusing, shear with a principal axis in the direction of the wave-vector projection into the source plane (φ), and no rotation. The $\times$ polarization produces no focusing, a shear comparable in strength to that of the $+$ polarization but with principal axis at 45°, and induces rigid rotation of the image. In other words, the $+$ polarization produces similar effects as scalar shear, while the $\times$ polarization induces truly pseudo-scalar shear (the change from a right-handed coordinate system to a left-handed one changes the sign of $h_\times$ but not of h_+).

We now use the relation $2(1-\mu)\beta + (1-\mu)^2\alpha = 1 - e^{ikz_s(1-\mu)}$ to express the rotation ω as

$$\omega = \frac{1}{2}(1+\mu)\Delta h_\times(\mathbf{k}) \tag{23}$$

where $\Delta h_\times(\mathbf{k}) = h_\times(\mathbf{k})(e^{ik(1-\mu)z_s} - 1)$ is the variation in the amplitude of the $\times$ component of the gravitational wave between the events of emission of the light ray at the source and its arrival to the observer at the origin. Clearly, there is no possible cumulative effect. The rotation can never exceed the dimensionless wave amplitude.

To separate the local distortion effects from those produced during light propagation we evaluate $\tilde{\psi}_{ab} = \psi_{ab} - \frac{1}{2}\Delta h_{ab}$, in this case the proper distortion matrix elements read:

$$\begin{aligned} \tilde{\kappa} &= 0 \\ \tilde{\gamma}_{(+,\times)} &= \tfrac{1}{2}(1-\mu)^2\alpha(\mu-1, z_s)h_{(+,\times)} \\ \tilde{\omega} &= \omega \quad . \end{aligned} \tag{24}$$

Expressed in terms of proper areas and as a function of affine distance to the source, there is no focusing to linear order in the gravity wave amplitude, in agreement with previous conclusions by Zipoy & Bertotti (1968).

Consider now a stochastic background of gravitational radiation, built as a superposition of plane sinusoidal waves. Contrary to the scalar case, they induce distortions just of the order of their dimensionless amplitude, but not larger (Kaiser & Jaffe 1997). Consider, for instance, an stochastic background such as that predicted by inflationary cosmological models (Abbott & Wise 1984), (rms amplitude proportional to the wavelength), with the largest possible amplitude compatible with the observed CMB anisotropy. The rms amplitude is of the order of $h(k) \sim 10^{-6} H_o/k$. The dominant effect is now that of the wavelengths comparable to the distance to the source. Distortions of order $\gamma \simeq 10^{-6}$, a factor 10^{-4} smaller than the effect expected to be induced by scalar perturbations, are unlikely to be detected. At any rate, it is interesting to observe that there are specific footprints of weak lensing by tensor perturbations, due to its pseudo-scalar contribution, which may eventually serve to distinguish it from that produced by density inhomogeneities.

3. Rotation of the plane of polarization by gravity waves

One specific footprint of weak lensing by gravity waves is rotation, which is not induced by scalar perturbations. Rotation of an image may be difficult to ascertain, without a knowledge of the intrinsic source orientation. The polarization properties of the electromagnetic emission of the source may, in some cases, bear some correlation with the intrinsic source orientation. The angle between some structural direction in a source and the direction of the plane of polarization is thus potentially a useful tool to search for eventual image rotation by weak lensing. Since the plane of polarization may also rotate as electromagnetic radiation propagates across density inhomogeneities or gravitational waves, we should also estimate this effect. To do so, in this section we work within the geometric optics approximation of the solution to Maxwell equations in curved space-time (Misner, Thorne, & Wheeler 1973), which imply that the direction of linear polarization is parallel-transported along the photon path. It is intuitive that scalar perturbations, which have

no handedness, do not rotate the plane of polarization. In the case of gravity waves, however, we may expect them to induce a "gravitational Faraday effect" (Perlick & Hasse 1993, Cooperstock & Faraoni 1993). Image rotation being a consequence of the geodesic deviation between neighboring rays while polarization rotation arises from parallel transport, we may in principle expect them to be different, and thus potentially provide a test to detect absolute rotations.

Typical wavelenghs of electromagnetic radiation are extremely short compared to the radius of curvature of space-time and to the typical length over which the amplitude, polarization, and wavelength of the electromagnetic waves may vary. Thus, we may confidently apply the geometric optics approximation in our analysis, and electromagnetic radiation may be regarded locally as plane waves that propagate through a space-time of negligible curvature. In the Lorentz gauge the electromagnetic vector potential may be written as

$$A^\mu = \Re\{a^\mu e^{i\theta}\} \tag{25}$$

where a^μ is a vector amplitude and θ is a real phase that rapidly changes, with the typical frequency of the electromagnetic radiation, $\theta = \omega p^\alpha x_\alpha + \text{const.}$, with ω the wave frequency and p^α the dimensionless wave-vector. Expanding the solution of Maxwell's equations in powers of the electromagnetic wavelength divided by the length-scale over which the gravitational potentials vary significantly (the gravitational wave wavelength in our case) one finds to lowest order that the photon paths are null geodesic, and to the next order that the vector amplitude a^μ can be written as $a^\mu = af^\mu$ where a is a scalar amplitude and f^μ is the polarization vector (normalized as $f^\mu f_\mu = 1$), which satisfies the equation of parallel transport along the photon path.

The electric field E^j measured by a comoving observer is derived from the electromagnetic field tensor $F^{\mu\nu} = A^{\nu;\mu} - A^{\mu;\nu}$ as

$$E^j = F^{0j} = \Re\{[(a^{j,0} - a^{0,j}) + i\omega(a^j p^0 - a^0 p^j)]e^{i\theta}\} \quad . \tag{26}$$

The length-scale over which the vector amplitude varies being much longer than the electromagnetic wavelength, the first term in the r.h.s. of Eq.(26) is negligible compared to the second. Thus, the direction of the electric field is indeed given by the direction of a^i, in turn determined by f^i.

We shall calculate the projection of the electric field direction into the plane perpendicular to the x^3-axis, f^a with $a = 1, 2$. Even though the photon paths deviate away from the polar axis in the presence of metric fluctuations, with typical deflections of order h, the projection into the plane perpendicular to unperturbed photon paths differs from the projection into the plane transverse to the actual photon trajectory by terms of order h^2.

The transverse components of the polarization vector verify the parallel-transport equation, which along a ray that propagates nearly parallel to the x^3-axis towards the origin read:

$$\frac{df^a}{dz} = \frac{1}{2}\delta^{ab}(h_{bc,0} - h_{bc,3} + h_{3c,b} - h_{3b,c})f^c \quad . \tag{27}$$

Notice that, to the order considered, the difference between the affine parameter z and the x^3-coordinate is now irrelevant.

In the case of scalar static perturbations, $h_{ab,3} = h_{,3}\delta_{ab}$ is the only non-zero term in the r.h.s. of this equation. The solution to eq. (27) is such that

$$f^a(z=0) = [\delta^a{}_b + \frac{1}{2}\Delta h^a{}_b] f^b(z_s) \quad . \tag{28}$$

Since Δh_{ab} is diagonal, f^1 and f^2 are modified at the same rate, and thus there is no rotation of the plane of polarization with respect to the fixed coordinate grid. In other words: if

$$\chi(z) = \arctan\left(\frac{f^2(z)}{f^1(z)}\right) \tag{29}$$

and $\Delta\chi = \chi(z_s) - \chi(0)$, then

$$\Delta\chi = 0 \quad \text{(scalar)} \quad . \tag{30}$$

In the presence of a plane and monochromatic gravitational wave with wave-vector $\mathbf{k} = k\mathbf{m}$, eq. (27) has instead non-diagonal terms

$$\frac{df^a}{dz} = \frac{-ik}{2} e^{ikz(1-\mu)} [h_3{}^a(\mathbf{k}) m_b - h_{3b}(\mathbf{k}) m^a + (1-\mu) h^a{}_b(\mathbf{k})] f^b \quad . \tag{31}$$

Integration of this equation between the source and the observer leads to

$$f^a(z=0) = [\delta^a{}_b - \omega\epsilon^a{}_b + \frac{1}{2}\Delta h^a{}_b] f^b(z_s) \tag{32}$$

where, as in the previous section, $\Delta h_{ab} = h_{ab}(z_s, t_e) - h_{ab}(0, t_o)$ denotes the variation in the gravitational wave amplitude between the events of emission and observation of the photons, and $\omega = \frac{1}{2}(1+\mu)\Delta h_\times$ coincides with the value for the image rotation induced by gravity waves. The angle that the polarization forms with respect to the x^1 -axis in this fixed coordinate grid thus changes by the amount

$$\Delta\chi = \omega + \frac{1}{4}(1+\mu^2)\Delta h_+ \sin 2(\varphi - \chi) + \frac{1}{2}\mu\Delta h_\times \cos 2(\varphi - \chi) \quad \text{(tensor)} \quad . \tag{33}$$

The first term in the r.h.s, independent of the initial direction of polarization, coincides with the image rotation ω derived in the previous section. The other two terms, that arise from the presence of Δh_{ab} in eq. (32), coincide with the apparent rotation of a direction that subtends an angle χ with respect to the x^1-axis as a consequence of the local variation in the gravitational potentials between the source and observer locations (as we shall discuss further in the next section).

Clearly, the rotation of the plane of polarization does not accumulate over distances longer than the wavelengths as light travels through a stochastic background of gravitational waves, since the effect is proportional to the change in the wave amplitude between the emission and observation events. The rotation of the plane of polarization produced by a stochastic background of gravitational waves of cosmological wavelengths and amplitude of order 10^{-6} is thus totally negligible.

4. Apparent rotations

The rotation of the plane of polarization as light propagates across gravity waves derived in the previous section, or the lack of it in the case of scalar perturbations, were measured with respect to the fixed coordinate grid (x^1, x^2, x^3). Notice, however, that due to weak lensing shear and rotation, be it of scalar or tensor nature, light originated in a point located at $\bar{\theta}^a z_s$, in a direction that forms an angle $\xi = \arctan(\bar{\theta}^2/\bar{\theta}^1)$ with respect to the x^1-axis in the fixed coordinate grid, is seen by the observer at the origin as coming from a direction, in this same coordinate grid, at an angle $\xi + \Delta\xi = \arctan(\theta^2/\theta^1)$, with

$$\Delta\xi = \omega + \gamma \sin 2(\phi - \xi) \quad . \tag{34}$$

We recall that γ is the shear intensity and ϕ its direction. A given fixed direction at the source is seen by the observer not just rotated by ω with respect to the absolute direction in this Euclidean grid, but is also affected by shear. In the case of scalar fluctuations, the apparent rotation of a fixed direction ξ induced by a single Fourier mode is

$$\Delta\xi = \tilde{\gamma} \sin 2(\varphi - \xi) \quad \text{(scalar)}. \tag{35}$$

In the case of gravity waves, it is given by

$$\begin{aligned} \Delta\xi = \; & \omega + \tfrac{1}{4}(1+\mu^2)\Delta h_+ \sin 2(\varphi - \xi) + \tfrac{1}{2}\mu \Delta h_\times \cos 2(\varphi - \xi) \\ & + \tilde{\gamma}_+ \sin 2(\varphi - \xi) - \tilde{\gamma}_\times \cos 2(\varphi - \xi) \qquad \text{(tensor)}. \end{aligned} \tag{36}$$

The first term in the r.h.s. of this equation is the rigid rotation ω. The next two terms are the effect upon a given fixed coordinate direction of the change in the gravitational potential between the emission and observation events. The rest is the additional apparent rotation due to the proper shear $\tilde{\gamma}$, given by eq. (24).

The question to address is what are the potentially observable consequences of apparent rotations of directions in the source relative to its plane of polarization. A possible approach is to consider sources with some structural, intrinsic direction. We can then answer the question of what happens to the apparent angle between the plane of polarization and that structural direction as light from the source propagates across metric perturbations.

Consider sources with a significant degree of linear polarization and elongated shape, with approximately elliptical isophotes. For these objects we denote with ψ the position angle on the sky (the orientation of the major axis of the image), and with χ the direction of the plane of integrated linear polarization, after subtraction of the wavelength-dependent Faraday rotation along intervening magnetic fields.

Weak lensing by metric perturbations produces focusing, shear, and rotation. Convergence amplifies sources without change in their shape, so it does not affect the observed position angle ψ. Rotation, as described by ω in eq. (12), accounts for a rigid rotation of the image, with the distance between any pair of points unchanged. Tensor perturbations rigidly rotate the position

angle ψ by the amount ω. However, even when no rigid rotation occurs, as is the case for scalar perturbations, shear distorts a source image in such a way that an ellipse is still seen as an ellipse but with different ellipticity and orientation.

Consider an elliptical source whose principal axis forms an angle ψ with respect to the x^1-axis in the source plane. $\bar{x}^a = \bar{\theta}^a z_s$ is taken to describe such an ellipse. We denote the length of the major and minor axes by a and b respectively. We define the ellipticity of the source as $\epsilon = (a^2 - b^2)/(a^2 + b^2)$. When light from this object arrives to the observer distorted by convergence, shear, and rotation, the image parametrization $x^a = (\delta^a{}_b + \psi^a{}_b)\bar{\theta}^b z_s$ in the observer sphere still describes an ellipse, but rotated an angle $\Delta\psi$ and with a different ellipticity ϵ', given by (if the distortion is small):

$$\tan(2\Delta\psi) = \frac{2[\omega + (\gamma/\epsilon)\sin 2(\phi - \psi)]}{1 + 2(\gamma/\epsilon)\cos 2(\phi - \psi)} \tag{37}$$

$$\epsilon' = \frac{\sqrt{[\epsilon + 2\gamma\cos 2(\phi - \psi)]^2 + 4[\epsilon\omega + \gamma\sin 2(\phi - \psi)]^2}}{1 + 2\epsilon\gamma\cos 2(\phi - \psi)} \quad . \tag{38}$$

Here ϕ is, as in previous sections, the direction of the shear principal axis. If the ellipticity is larger than the shear, $\epsilon \gg \gamma$, we can approximate

$$\Delta\psi \approx \omega + \frac{\gamma}{\epsilon}\sin 2(\phi - \psi) \tag{39}$$

(which of course coincides with eq. (34) in the limit $\epsilon = 1$), and the new ellipticity is

$$\epsilon' \approx \epsilon + 2\gamma(1 - \epsilon)\cos 2(\phi - \psi) \quad . \tag{40}$$

In the opposite limit, of negligible eccentricity, $\epsilon' \approx 2\gamma$ and $\Delta\psi \approx \phi$, which simply means that the nearly circular source is distorted in the direction of the shear.

We conclude that the angle between the plane of integrated linear polarization and the source axis, $\chi - \psi$, is observed to change, as light propagates across density inhomogeneities and gravitational waves, by the difference $\Delta(\chi - \psi)$ as given by eqs. (30,33) and (37).

Rotation and shear produced by a cosmological background of gravitational waves, at most of order 10^{-6}, lead to a totally negligible apparent rotation between the plane of polarization and the source position angle.

Scalar shear leads to an apparent rotation $\Delta(\chi - \psi) = -\Delta\psi$, since $\Delta\chi = 0$ for scalar perturbations. In sources with large ellipticity ($\epsilon \approx 1$), the apparent rotation is of the order of the shear itself. Such a small apparent rotation, at most of the order of a few degrees, is not likely to have significant observational consequences. Gravitational distortions due to weak lensing by large-scale structure will produce no observable effect on the polarization properties of extended radio jets in quasars. In the opposite limit, nearly circular sources appear distorted in the direction of the scalar shear, while their polarization plane remains in the same direction. Scalar shear

is thus able to mask any correlation between orientation and polarization that a nearly circular source might have.

The most interesting regime may be the case of sources with ellipticities of the order of $\epsilon \approx 0.1$. These are the smallest ellipticities among radio sources that still present some significant (larger than 5%) degree of linear polarization. They can display an apparent rotation of the plane of polarization relative to their major axis of order

$$\Delta(\chi - \psi) \approx \frac{\gamma}{\epsilon} \quad . \tag{41}$$

A scalar shear of 1% can change the observed angle between polarization and source orientation by 5 degrees.

5. Effects of shear upon the correlation between source polarization and orientation

The direction of the plane of polarization in radio galaxies and quasars is expected to trace the geometric configuration of the magnetic field in such objects, at least in sources with a significant degree of linear polarization. Discrepancies between the observed values and expectations based in models for the magnetic field in distant radio sources could eventually signal the presence of unusual effects additional to the standard Faraday rotation in the polarization properties of light as it propagates through the universe (Birch 1982, Carroll, Field & Jackiw 1990, Harari & Sikivie 1992, Nodland & Ralston 1997, Carroll & Field 1997).

It is possible that weak lensing by scalar perturbations explain the dispersion in the direction of integrated polarization of cosmologically distant radio sources away from the perpendicular to their major axis. We now discuss this possibility.

Consider the data on 160 radio sources used in Carroll et. al (1990), with the corrections noted in Carroll and Field (1997). There are 89 sources with redshift $z < 0.3$ and 71 with redshift $z > 0.3$. The histogram of number of radio sources vs. $\chi - \psi$ in the case of nearby ($z < 0.3$) galaxies does not display a very strong correlation, although there is a narrow enhancement around $\chi - \psi \approx 0°$ and a broader peak at $\chi - \psi \approx 90°$. In the case of distant ($z > 0.3$) sources, the peak around $\chi - \psi \approx 90°$ enhances dramatically, while there is no noticeabe peak around $\chi - \psi \approx 0°$. It is likely that any correlation between polarization and source orientation be weaker in the case of sources only weakly polarized. Restriction of the sample to the case of sources with maximum polarization $p_{\rm max} > 5\%$ does indeed significantly enhance the two peaks in the histogram of nearby sources, while the peak around $\chi - \psi \approx 90°$ for distant sources becomes even more convincing.

The most likely explanation for these observations is that the sample consists of two different populations. One population would consist of low luminosity sources, for which either there is no significant correlation between polarization and position angle, particularly in the case of weakly polarized sources, or have $\chi - \psi \approx 0°$. The low luminosity population would be underepresented in

the high redshift observations, where the second class, of high luminosity sources with polarization almost perpendicular to their axis, would dominate (Carroll & Field 1997, Clarke et al. 1980).

We now put forward the following scenario, which appears to be consistent with present observations. Assume that most of the distant radio sources that have a significant degree of integrated linear polarization are polarized in a direction almost perpendicular to their major axis (say less than 5° away from it), while a small fraction does not posses a significant correlation between polarization and position angle. Scalar shear then acts stochastically upon the light emitted by these sources. The result should be an increase in the spread of the observed $\chi - \psi$ away from 90°, proportional to the rms value of the shear. The effect actually depends upon the source redshift and ellipticity. If the number of data points were sufficiently large one could search for a correlation between departures of $\chi - \psi$ away from 90° with the source redshift and ellipticity, and put to test this scenario. Present observations are not sufficient to make serious attempts in this direction, but we shall argue that they are at least consistent with the above scenario, if the scalar shear has rms value around 5%. Notice that this shear needs to be coherent only over small scales, of the order of the angular size of the source.

Given that the number of data points is not sufficiently large to search for any correlation of the effect of shear with the source redshift and ellipticity, we shall estimate the effect to be expected in a population of radio sources with ellipticity equal to the mean in the sample, at a redshift equivalent to the mean redshift. Since shear effects do not significantly accumulate in the case of nearby sources, we choose to consider just the 51 sources with $z > 0.3$ and maximum integrated linear polarization $p_{\rm max} > 5\%$. We plot the corresponding histogram in Figure 1. These sources have a mean value of $\chi - \psi$ equal to 90° with rms dispersion $\sigma = 33^\circ$, and mean redshift $\bar{z} = 0.85$. Of these 51 sources, 41 have their angle of intrinsic polarization less than 45° away from the direction perpendicular to the source axis, while the remaining 10 appear to be more or less uniformly distributed at angles less than 45° away from the source axis, although the number is too small to draw significant conclusions. If we exclude from the sample these 10 sources, under the assumption that they may be predominantly members of a population with weak correlation between polarization and position angles, the remaining 41 sources have a mean value of $\chi - \psi$ of 91° with rms dispersion $\sigma = 19^\circ$ and mean redshift $\bar{z} = 0.84$. We have evaluated the ellipticites of all but 4 of them, for which we found the values of the major and minor axis quoted in Gregory at. al (1996). The mean ellipticity of the sample is $\bar{\epsilon} \approx 0.15$. We put forward the hypothesis that the intrinsic dispersion of $\chi - \psi$ away from 90° is much smaller than the observed value, the observed dispersion being the consequence of the action of stochastic cosmic shear.

We now estimate the amount of shear necessary to produce a mean square rotation $\sigma \approx 20^\circ$ upon a population of sources with ellipticity $\epsilon = 0.15$, assumed to be located at equal redshift. In the limit $\bar{\gamma}/\epsilon << 1$, where $\bar{\gamma}$ is the rms value of the shear, assumed to act in all directions in the plane perpendicular to the photon paths with uniform probability, the results of the previous

section (eq. (39)) indicate that the rms value of the apparent rotation is

$$\sigma_\psi \equiv < (\Delta\psi)^2 >^{1/2} \approx \left[\frac{2}{\pi}\int_0^{\pi/2} d\phi \left(\frac{\bar{\gamma}}{\epsilon}\right)^2 \sin^2 2\phi\right]^{1/2} = \frac{1}{\sqrt{2}}\frac{\bar{\gamma}}{\epsilon} \quad . \tag{42}$$

A value $\sigma_\psi = 0.35$ (20°) corresponds to $\bar{\gamma}/\epsilon \approx 0.5$, which implies a rms shear of 7.5% if the ellipticity of the sources is $\epsilon = 0.15$. Eq. (39), however, underestimates the apparent rotation when γ/ϵ is not significantly smaller than unity, as is frequently the case if the rms value is $\bar{\gamma}/\epsilon = 0.5$. To better estimate the rms apparent rotation in a general case we should use eq. (37). Assuming a gaussian distribution of γ around a zero mean, with rms dispersion $\bar{\gamma}$, and uniform probability distribution in the shear direction ϕ, the rms rotation reads

$$\sigma_\psi = \left[\frac{2}{\pi}\int_0^{\pi/2} d\phi \int_{-\infty}^{+\infty} d\gamma \frac{1}{4} \arctan^2\left(\frac{2(\gamma/\epsilon)\sin 2\phi}{1+2(\gamma/\epsilon)\cos 2\phi}\right)\frac{e^{-\gamma^2/2\bar{\gamma}^2}}{\sqrt{2\pi}\bar{\gamma}}\right]^{1/2} \quad . \tag{43}$$

In the limit $\bar{\gamma}/\epsilon << 1$, this expression coincides with the approximate value of eq. (42). In the opposite limit, $\bar{\gamma}/\epsilon >> 1$, it tends to $\pi/(2\sqrt{3})$ ($\approx 52^\circ$), which corresponds to a uniform probability distribution for rotations between 0 and $\pi/2$, as expected since in this limit the apparent rotation is given by $\psi = \phi$.

The result of the numerical evaluation of eq. (43) is plotted in Figure 2. It indicates that $\sigma_\psi = 0.35$ (20°) when $\bar{\gamma}/\epsilon \approx 0.3$, which if $\epsilon \approx 0.15$ implies

$$\bar{\gamma} \approx 0.05 \quad . \tag{44}$$

The statistical significance of the limited sample analysed, as well as the crude approach taken towards the redshift and ellipticity dependence of the effect, make this result only an approximate estimate. It seriously indicates, however, that the observed departures of the direction of intrinsic polarization away from the perpendicular to the source axis in distant radio sources are consistent with the hypothesis that their origin is the effect of shear with rms value around or below 5%. Much larger values of $\bar{\gamma}$ would actually conflict with the observed correlation between source polarization and orientation.

Other accurate measurements of the polarization properties of distant sources come from high resolution measurements of highly polarized local emission regions (Leahy 1997, Wardle et al. 1997). In these observations, a very tight correlation is found between the plane of polarization and some structural direction (such as the direction of intensity gradients), in agreement with theoretical expectations. Shear by weak lensing effects would rotate the observed angle between polarization and that locally defined direction by the rms amount $\Delta\psi \approx \bar{\gamma}/\sqrt{2}$ only.

6. Conclusions

Measurement of the rotation of the intrinsic polarization with respect to the local orientation of each jet segment in extended radio jets has been used to probe the mass distribution in

intervening lensing galaxies (Kronberg et al. 1991, Kronberg, Dyer, & Röser 1996). In this paper we extended this method of gravitational “alignment-breaking” to the case where the change in the observed polarization properties of distant radio sources is due to the weak lensing effect produced by large scale density inhomogeneities. Indeed, we argued that cosmic shear can have a significant impact upon the observed angle $\chi - \psi$ between the direction of linear polarization and some structural direction of radio sources. In extended radio sources (sources with large ellipticities $\epsilon \approx 1$) we found that weak lensing effects are negligible, since the change in the relative angle $\chi - \psi$ does not exceed the magnitud of the cosmic shear. On the other hand, radio sources with intermediate ellipticities, of the order of $\epsilon \approx 0.1$, can display a significant apparent rotation of the plane of polarization relative to their major axis due to gravitational distortions by large-scale structure.

Our study provides an alternative approach to evidence the effects of cosmic shear coherent over small angular scales, of the order of the source angular diameter. This approach is complementary to methods currently used to detect cosmic shear coherent over the extension of galactic fields through the search of correlations in the ellipticities of the galaxies.

Our main conclusion is that density inhomogeneities do not rotate the direction of linear polarization, but since they distort elliptical shapes, the angle between the direction of linear polarization and the source major axis observed from Earth differs from the angle at the source (after subtraction of Faraday rotation). In the limit $\gamma << \epsilon$, where γ is the value of the shear induced by the perturbations between the source and the observer, and ϵ is the source ellipticity, the apparent rotation is of order $\Delta\psi \approx \frac{\gamma}{\epsilon}$. This becomes very interesting in the case of sources with ellipticity of order $\epsilon \approx 0.1$, since it implies, in the presence of a shear of order 1 %, a rotation of around 5 degrees, already comparable or larger than the typical errors of a few degrees in Faraday rotation subtraction.

We have shown that the observed correlation between polarization properties and source orientation in distant radio galaxies and quasars can be used to either constrain or eventually reveal the presence of cosmic shear. The number of present observations does not appear to be large enough to test with sufficient statistical significance whether the observed departures of the intrinsic polarization angle away from the direction perpendicular to the source axis may be ascribed to the effects of scalar shear. Present observations show that the largest fraction of distant radio sources have integrated linear polarization in a direction almost perpendicular to the source axis, with a dispersion around the mean of about 20°. We have argued that current observations are consistent with the hypothesis that this dispersion is caused by a shear with rms value around 5%, acting upon a population of radio sources that has a much smaller intrinsic dispersion. A rms shear much larger than 5% would induce rms rotations much larger than 20°, and would thus be inconsistent with the observed correlation between source polarization and orientation. With a larger number of observations available, the cosmic shear hypothesis could be tested through statistical correlations between the polarization vs. orientation angle and the source redshift and ellipticity.

Finally, weak lensing by a cosmological background of gravity waves is mostly of academic interest, since their effect does not accumulate with distance as that of scalar perturbations, and thus the expected distortions are completely negligible (of order 10^{-6}). From an academic standpoint, however, it is interesting to emphasize the differences between tensor and scalar distortions. Gravity waves produce no focusing to linear order in the metric perturbation. They do induce shear plus rigid rotation of images, and also rotate the plane of polarization of electromagnetic radiation.

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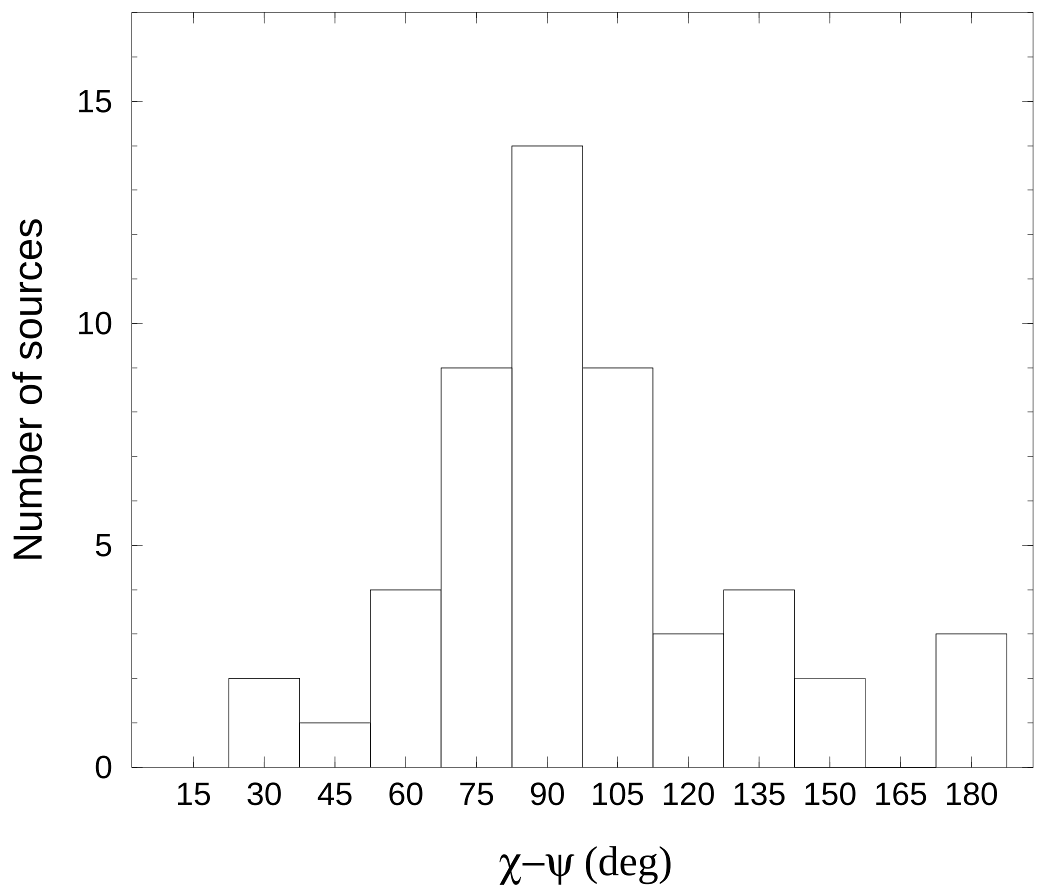

Fig. 1.— Histogram of number of sources vs. $\chi - \psi$ (polarization - position angle) for the 51 sources with redshift $z > 0.3$ and maximum integrated polarization $p_{\max} > 5\%$

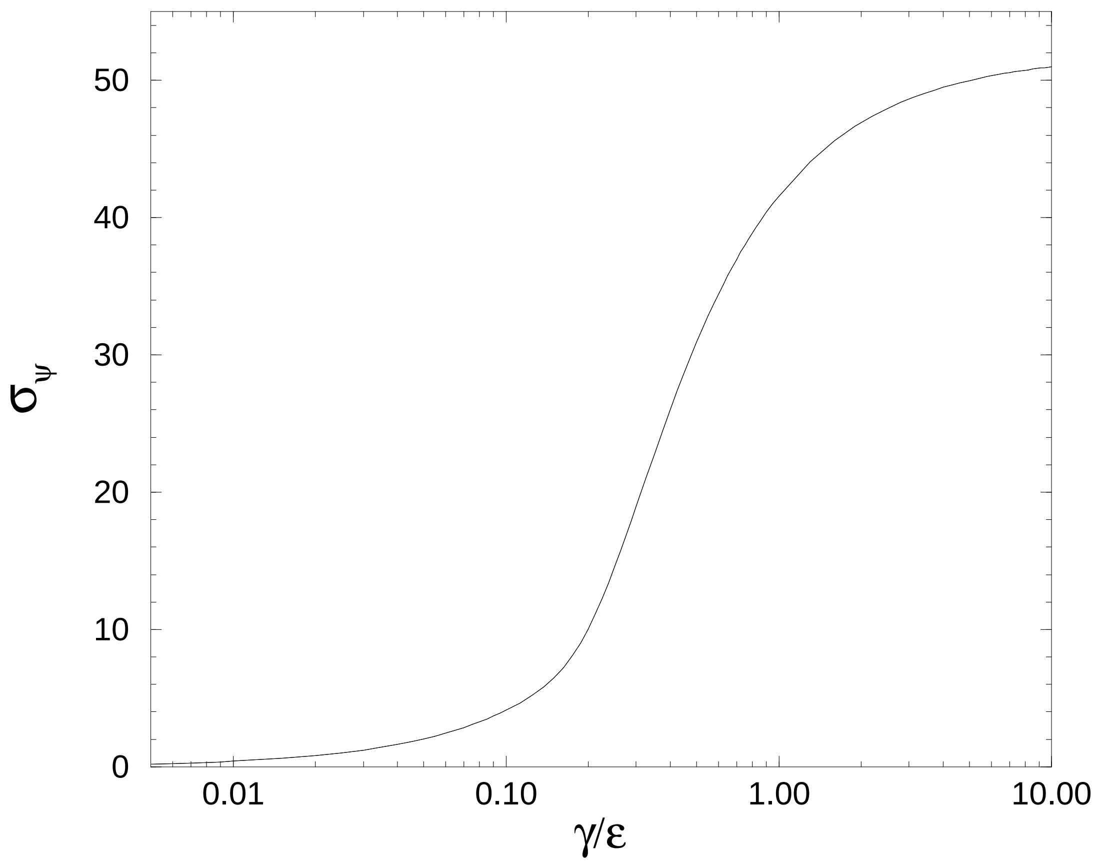

Fig. 2.— Root mean square value of the apparent image rotation σ_ψ induced by a rms shear $\bar{\gamma}$ upon a source with ellipticity ϵ, plotted as a function of $\bar{\gamma}/\epsilon$